Instructor:	Dr. Kyle Kinneberg	**Time:**	MWF 2–2:50PM
Office:	Herman Brown Hall, 452	**Classroom:**	Herman Brown Hall, 423
Email:	kyle.kinneberg@rice.edu	**Office hours:**	By appointment

Class webpage: OwlSpace page.

Description: The three higher-dimensional versions of the fundamental theorem of calculus – Green's theorem, the divergence theorem, and Stokes' theorem – that one encounters in a typical multi-variable calculus course have two paradoxical characteristics. On the one hand, these results lie at the center of many areas of mathematics and the physical sciences, including analysis of partial differential equations, fluid mechanics, electromagnetism, and thermodynamics. On the other hand, most students don't understand these theorems and almost certainly forget them after the final exam (I do not except myself from this category).

The purpose of this course is essentially a semester-long study of the modern version of Stokes' theorem and the mathematics needed to build it up (Green's theorem and the divergence theorem are corollaries of Stokes). In fact, the mathematics we develop along the way will include topics that appear at the foundation of modern differential geometry and algebraic topology.

The first part of the course will focus on calculus in higher dimensions: differentiation and integration of functions defined on $\mathbb{R}^n$, along with a basic study of topology in $\mathbb{R}^n$. We will see, for example, the implicit and inverse function theorems, Fubini's theorem, and the change-of-variables formula for integration.

The second part of the course will introduce important algebraic objects that are used in geometry (e.g., tensors, differential forms, and singular chains), along with operations between them (e.g., products, exterior differentiation, and boundary chains). This will be done in the setting of $\mathbb{R}^n$, and we will see a version of Stokes' theorem here.

The third part of the course will move to the setting of manifolds. A manifold is, colloquially, a space that, on small patches, "looks like $\mathbb{R}^n$" (e.g., a sphere, or the surface of a donut, or the entire donut when we allow for manifolds-with-boundary). Formally, it is helpful to keep the following in mind: calculus can be done in $\mathbb{R}^n$, calculus involves *local* analysis of functions, manifolds look like $\mathbb{R}^n$ *locally*...therefore, calculus can be done on manifolds! We will make this rigorous by transferring our work from the second part of the course to the manifold setting. In the end, we will prove the following general version of Stokes' theorem and understand what it actually means!

$$\int_M \mathrm{d}\omega = \int_{\partial M} \omega$$

Textbook: We will follow Michael Spivak's book "Calculus on Manifolds" very closely. This short textbook was written precisely for a course like this, and his writing is quite clear. This is the only required textbook.

If you find Spivak's book to be too light on background, details, or examples, you can consult James Munkres' book "Analysis on Manifolds," which covers almost the same topics but in more than twice the length.

Homework: I will assign homework weekly, due on Wednesdays. Most exercises will be taken from Spivak's book. I will post the assignments on OwlSpace but all submissions will be non-electronic.

Homework assignments are not pledged, and I encourage you to work together, exchanging ideas on the exercises. However, the write-up that you turn in must be done independently and should

reflect your understanding of the material. This also means that the use of solution manuals or online solutions is not allowed.

Most of the homework exercises will be proofs. Consequently, your homework grades will be based on both correctness and clarity of the solutions. To aid in clarity, I suggest the following: Briefly state the goal of the problem; Justify each logical step; Present your solution in a neat format that is easy to read. In particular, do not turn in scratch work.

Late homework assignments will not be accepted. If you will not be able to attend lecture the day an assignment is due, please make arrangements to turn it in early. I will, however, drop your lowest homework score.

Exams: There will be one midterm exam and one comprehensive final exam. Both will be take-home. I will hand out the midterm on February 25, and it will be due at the beginning of class on February 27. In accordance with Rice policy, the final exam will be due at the end of the scheduled exam period for our class, which is not determined yet. However, it is the policy of the Rice Mathematics Department that no final may be given early to accommodate student travel plans.

Exams will be pledged, and the use of books, notes, or online sources is not allowed. If you have a conflict with the exam dates (a holiday that you observe, interviews, etc.), you must let me know at least a week in advance.

Any complaints or requests concerning the grading of exams must be made within one week after receiving back the graded exam.

Evaluation: Your final grade will be determined as the maximum of the following:

20% Homework + 35% Midterm exam + 45% Final exam.
20% Homework + 30% Midterm exam + 50% Final exam.

Expectations: Mathematics is learned by grappling with the material for yourself through reading and doing many exercises. Simply attending lectures will not be sufficient for understanding. I encourage you to read supplementary material and solve exercises from sources other than homework sets. In a similar vein, though you should use office hours to clarify any topics that confuse you, they are also intended for further discussion on material we cover in lecture. As noted above, much of the theory that we will see in class lies at the root of very beautiful current mathematics, so there is a great deal more that one can say about it.

One warning: in reading mathematics, statements of results must be "deceiphered," turning mathematical symbols and notation back into ideas and concepts. This process can be especially difficult in differential geometry/topology, where there are some symbols that have different meanings, depending on the context (e.g., ∂ or $\mathrm{d}x$), and there are some concepts that have multiple options for notation (e.g., partial derivatives). Often, you will find, the underlying concept is not difficult to grasp, as long as you have the patience to wade through the mathematical notation to get to it.

Disability Support: Any student with a documented disability who is seeking academic adjustments or accommodations should speak with me during the first week of class. Such discussions will remain confidential, but the student will also need to contact Disability Support Services in the Allen Center.